On Infinite-Horizon Sensor Scheduling

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Abstract

In this paper we consider the problem of infinite-horizon sensor scheduling for estimation in linear Gaussian systems. Due to possible channel capacity, energy budget or topological constraints, it is assumed that at each time step only a subset of the available sensors can be selected to send their observations to the fusion center, where the state of the system is estimated by means of a Kalman filter. Several important properties of the infinite-horizon schedules will be presented in this paper. In particular, we prove that the infinite-horizon average estimation error and the boundedness of a schedule are independent of the initial covariance matrix. We further provide a constructive proof that any feasible schedule with finite average estimation error can be arbitrarily approximated by a bounded periodic schedule. We later generalized our result to lossy networks. These theoretical results provide valuable insights and guidelines for the design of computationally efficient sensor scheduling policies.

Keywords: Wireless Sensor Networks, Kalman Filtering, Riccati Equation

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1. Introduction

Sensor networks span a wide spectrum of applications, e.g., environment and habitat monitoring, health care, home and office automation, and traffic control [1]. In many of these applications, a centralized fusion center is implemented to collect and process the measurements for estimation purposes. Sensor nodes are typically battery powered, and therefore energy constrained. Furthermore, their radios are low-power and may be subject to interference and fading. As a result of the bandwidth and energy constraints, it is not advisable, and sometimes infeasible, for all the sensors to communicate with the fusion center within each sampling period. Thus, it is of significant interest to determine sensor scheduling policies able to tradeoff energy/bandwidth consumption and estimation quality.

Sensor network energy consumption minimization and lifetime maximization problems have been active areas of research in recent years. Sensor networks energy management is typically carried out via efficient MAC protocols [2] or via efficient scheduling of sensor states [3, 4]. Xue and Ganz [5] showed that the lifetime of sensor networks is influenced by transmission schemes, network density and transceiver parameters with different constraints on network mobility, position awareness and maximum transmission ranges. Chamam and Pierre [6] proposed a sensor scheduling scheme capable of optimally putting sensors in active or inactive modes. Shi et al. [7] considered sensor energy minimization as a mean to maximize the network lifetime while guaranteeing a desired quality of the estimation accuracy. The same authors further proposed a sensor tree scheduling algorithm [8] which leads to longer network lifetimes.

Performance optimization for sensor networks under given energy constraints, which can be seen as the dual problem of network energy minimization, has also been studied by several researchers. Such constrained optimization problem has been studied for continuous-time linear systems by Miller and Runggaldier [9] and Mehra [10]. Krishnamurthy [11] derived the optimal sensor scheduling for the estimation of a Hidden Markov Model based system. For discrete-time linear systems, approaches using dynamic programming [12], greedy algorithms [13], convex optimization [14, 15, 16] and branch and bound [17] have been proposed to find the optimal or suboptimal sensor scheduling over finite time horizons. In general, the sensor scheduling problem is a combinatorial optimization problem [18] and thus the exact optimal solution over long time horizons is computationally intractable. However, the exact optimal schedule can be computed in some very particular cases. For instance, Shi and Zhang [19] and Hovareshti et al.[20] prove that under certain conditions, the optimal infinite-horizon schedule is periodic for a system with two smart sensors.

Power control has also been studied [21, 22] to increase the energy efficiency of sensors. To this end, a sensor could use a lower power level to communicate information, which results in either a lower SNR, an increase in communication delay or a larger packet drop probability. Conceptually, for sensors with finitely many power levels, a virtual sensor could be assigned to each power level. Hence, the usage of power control can be seen as a special case of sensor scheduling.

In most of the works cited above, the optimal schedule can only be computed for linear systems over a finite-horizon, while only for specific systems an infinite-horizon policy can be derived. Moreover, for general systems, the solution is usually given as the result of an optimization problem and thus implicit. In this paper, we consider the problem of sensor scheduling for state estimation of linear LTI systems with Gaussian noise over an infinite-horizon. In particular we focus on proving several fundamental properties that can be used as guideline for the analysis and design of infinite-horizon sensor schedules. In particular, we prove the following two propositions concerning scheduling policies:

1. The average estimation error of a schedule is independent of the initial covariance of x_0.
2. Any schedule that has a bounded average estimation error can be arbitrarily approximated (both in terms of average estimation error and communication rate) by bounded periodic schedules.

These results have important practical consequences as bounded periodic schedules are easier to compute than general ones.

The rest of the paper is organized as follows: in Section 2, we formulate the infinite-horizon sensor scheduling problem. In Section 3, we prove that the average estimation covariance is independent of the initial conditions. We further provide a constructive proof that any feasible schedule can be arbitrarily approximated by bounded periodic schedules in Section 4. We then generalize our results to lossy networks in Section 5. A numerical example is presented in Section 6 to illustrate the performance of periodic schedules. Finally, Section 7 concludes the paper.

Notations

We summarize the notations used in this paper in Table 1.

S	Set of sensors
$\mathcal{S}$	Collection of all eligible subsets of S
$\mathcal{I}_k$	Subset of sensors selected at time k
σ	An infinite sensor schedule in the form of $(\mathcal{I}_1, \mathcal{I}_2, \dots)$
Σ	The covariance of the initial state x_0
Q	The covariance of process noise
R	The covariance of measurement noise
$J(\sigma, \Sigma)$	The average trace of the error covariance matrix
$r_i(\sigma)$	The average communication rate of sensor i

Table 1: Notations

2. Problem Formulation

Consider the following discrete-time LTI system

$$x_{k+1} = Ax_k + w_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ represents the state and $w_k \in \mathbb{R}^n$ the process noise. It is assumed that w_k and x_0 are independent Gaussian random vectors, $x_0 \sim \mathcal{N}(0,\ \Sigma)$ and $w_k \sim \mathcal{N}(0,\ Q)$, where $\Sigma, Q > 0$[2]. A wireless sensor network composed of m

[2]All the comparisons between matrices are in the sense of positive semidefinite.

sensing devices $S = \{s_1, \ldots, s_m\}$ and one fusion center is used to monitor the state of system (1). The measurement equation is

$$y_k = Cx_k + v_k, \tag{2}$$

where $y_k = [y_{k,1}, y_{k,2}, \ldots, y_{k,m}]' \in \mathbb{R}^m$ is the measurement vector[3]. Each element $y_{k,i}$ represents the measurement of sensor i at time k, $C = [C_1', \ldots, C_m']'$ is the observation matrix and the matrix pair (C, A) is assumed observable, $v_k \sim \mathcal{N}(0,\ R)$ is the measurement noise, assumed to be independent of x_0 and w_k.

Suppose that due to energy, bandwidth or topological constraints, only a subset of sensors can be chosen to send their measurements to the fusion center. Denote the collection of all eligible subsets as $\mathcal{S} \subseteq \mathcal{P}(S)$, where $\mathcal{P}(S)$ denotes the power set of S, i.e., the collection of all subsets of S.

For any $\mathcal{I} = \{s_{i_1}, \ldots, s_{i_l}\} \in \mathcal{S}$, we define the selection matrix $\Gamma(\mathcal{I})$

$$\Gamma(\mathcal{I}) \triangleq [e_{i_1}, \ldots, e_{i_l}]',$$

where e_i is the ith vector of the canonical basis, i.e. a vector with entries 0 everywhere, except a 1 at the ith entry. By means of this selection matrix we can define the matrices

$$C(\mathcal{I}) \triangleq \Gamma(\mathcal{I})C,\ R(\mathcal{I}) \triangleq \Gamma(\mathcal{I})R\Gamma(\mathcal{I})',$$

that allows one to define the matrix-valued function $g(X, \mathcal{I})$ as

$$g(X, \mathcal{I}) \triangleq \left[(AXA' + Q)^{-1} + C(\mathcal{I})'R(\mathcal{I})^{-1}C(\mathcal{I})\right]^{-1}.$$

A schedule is defined as an infinite sequence of $\sigma \triangleq (\mathcal{I}_1, \mathcal{I}_2, \ldots)$ satisfying the constraint $\mathcal{I}_k \in \mathcal{S}$. Clearly, if a schedule σ is used, the covariance of the Kalman filter satisfies the following equation:

$$P_k = g(P_{k-1}, \mathcal{I}_k),\ P_0 = \Sigma. \tag{3}$$

[3]The $'$ on a matrix always means transpose.

Remark 1. In case of quantized measurements, You et al. [23] and Msechu et al. [24] propose a Quantized Kalman Filtering (QKF) algorithm, where the approximated P_k follows a modified Riccati equation similar to (3). As a result, all the results discussed in this paper can be generalized to QKF.

Since P_k is a function of both the sensor schedule σ and the initial condition Σ, we will denote P_k as $P_k(\sigma, \Sigma)$. Let us define the cost function $J(\sigma, \Sigma)$ as

$$J(\sigma, \Sigma) \triangleq \limsup_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \mathrm{tr}(P_k(\sigma, \Sigma)).$$

$J(\sigma, \Sigma)$ can be seen as the average estimation error. Moreover, let us define the average communication rate of sensor i as

$$r_i(\sigma) \triangleq \limsup_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \mathbb{I}_{s_i \in \mathcal{I}_k},$$

where $\mathbb{I}$ is the indicator function.

Remark 2. Our formulation can address a large class of sensor selection problems. In particular, the set $\mathcal{S}$ can be used to characterize the topological and communication constraints of the network, while variables r_i define the average usage of the sensors, which can be used to define energy constraints on sensors.

We have the following definitions:

Definition 1. A schedule σ is called *feasible* if for all initial condition Σ, $J(\sigma, \Sigma) < \infty$.

Definition 2. A schedule σ is called *bounded* if for all initial condition $\Sigma > 0$, there exists a matrix $M(\Sigma)$, such that $P_k(\sigma, \Sigma) \le M(\Sigma)$ for all k[4].

Definition 3. A schedule σ is called *periodic* if there exists $T > 0$, such that $\mathcal{I}_{k+T} = \mathcal{I}_k$ for all k.

In the next section, we will prove that the cost function J does not depend on the initial condition Σ. In Section 4, we will prove that we can arbitrarily approximate a feasible schedule by means of bounded periodic schedules.

[4]Note that while boundedness clearly implies feasibility, the converse is not always true. It is enough to consider the sequence of P_k is $\{1, 0, 2, 0, 0, 3, 0, 0, 0, 4, , ...\}$ to see that while P_k is unbounded, the average cost $J(\sigma, \Sigma)$ would be bounded and in particular equals to 1.

3. Independence of the Initial Condition

This section is devoted to prove the following Theorem:

Theorem 1. *If there exists $\Sigma_0 > 0$ such that $J(\sigma, \Sigma_0) < \infty$, then for all $\Sigma > 0$, we have*

$$J(\sigma, \Sigma) = J(\sigma, \Sigma_0) < \infty.$$

Furthermore, if $P_k(\sigma, \Sigma_0) \leq M$ is bounded for all k, then the schedule σ is bounded.

Remark 3. The main consequence of Theorem 1 is that the cost is independent from the initial conditions but only depends on the scheduling, i.e. $J(\sigma, \Sigma) = J(\sigma)$ and that to check the *feasibility* [*boundedness*] of a schedule σ it is enough to check if $J(\sigma, \Sigma)$ $[P_k(\sigma, \Sigma)]$ is bounded for one initial condition $\Sigma > 0$ instead of all initial conditions.

To prove Theorem 1, a few intermediate results have to be derived. To this end, let us first define the functions:

$$h(X, Y) \triangleq X + Y, \tag{4}$$

where $X, Y \in \mathbb{R}^{n \times n}$ are positive semidefinite and

$$f(X, Y) \triangleq (X^{-1} + Y)^{-1}, \tag{5}$$

where $Y \in \mathbb{R}^{n \times n}$ are positive semidefinite and $X \in \mathbb{R}^{n \times n}$ are strictly positive definite[5]. Please note that these functions are linked to the function g as follows

$$g(X, \mathcal{I}) = f(h(A'XA, Q), C(\mathcal{I})'R^{-1}(\mathcal{I})C(\mathcal{I})). \tag{6}$$

Therefore, to prove the properties of g it is convenient to first study the functions f and h, whose main properties are summarized in the following Lemma:

Lemma 2. *For the functions $h(X, Y)$ and $f(X, Y)$ defined in* (4) *and* (5)*, the following propositions hold true:*

a) $f(X, Y)$ and $h(X, Y)$ are monotonically increasing with respect to X, i.e., if $X_1 \leq X_2$,

$$f(X_1, Y) \leq f(X_2, Y),\ h(X_1, Y) \leq h(X_2, Y). \tag{7}$$

[5] We require X to be strictly positive definite in order to ensure that f is a well-defined function.

b) The following inequalities hold:

$$f((1+\rho)X,Y) \leq (1+\rho)f(X,Y).\, \forall \rho \geq 0, \tag{8}$$
$$h((1+\rho)X,Y) \leq (1+\rho)h(X,Y).\, \forall \rho \geq 0. \tag{9}$$

c) if $Y \geq \alpha X$, then

$$h((1+\rho)X,Y) \leq \left(1+\frac{\rho}{1+\alpha}\right) h(X,Y).\, \forall \rho \geq 0. \tag{10}$$

A partial proof of Lemma 2 was reported in [15]. However, for the sake of completeness, the whole proof is included in this article.

Proof. *a)* The monotonicity of h and f can be proved by direct substitution in (4) and (5).

b) For (8), since $X > 0$ is strictly positive definite, there exists an invertible matrix $U \in \mathbb{R}^{n\times n}$ that can diagonalize X^{-1} and Y simultaneously, i.e.,

$$X^{-1} = U^{-1}(U^{-1})',\ Y = U^{-1}\Lambda(U^{-1})', \tag{11}$$

where $\Lambda = \operatorname{diag}(\lambda_1,\dots,\lambda_n)$ and $\lambda_i \geq 0$. Hence

$$\begin{aligned} &f((1+\rho)X,Y) - (1+\rho)f(X,Y) \\ &= U'\left\{\left[(1+\rho)^{-1}I+\Lambda\right]^{-1} - (1+\rho)(I+\Lambda)^{-1}\right\}U \\ &= U' \operatorname{diag}\left(\dots,\frac{1+\rho}{1+(1+\rho)\lambda_i} - \frac{1+\rho}{1+\lambda_i},\dots\right)U\,. \end{aligned}$$

For ρ positive the diagonal matrix in the equation above is negative semidefinite and (8) follows. For (9),

$$h((1+\rho)X,Y) - (1+\rho)h(X,Y) = ((1+\rho)X+Y) - (1+\rho)(X+Y) = -\rho Y \leq 0.$$

c) It is enough to substitute and obtain

$$\begin{aligned} &h((1+\rho)X,Y) - \left(1+\frac{\rho}{1+\alpha}\right)h(X,Y) = \\ &(1+\rho)X+Y - \left(1+\frac{\rho}{1+\alpha}\right)(X+Y) = \frac{\rho(\alpha X - Y)}{1+\alpha} \leq 0, \end{aligned}$$

which concludes the proof. □

The following Lemma illustrates the monotonicity and the "contraction" property of the Riccati equation g, which follows directly from Lemma 2 and the definition of g in terms of f and h given in (6):

Lemma 3. *For all $\rho \geq 0\,, X > X_0 > 0$, then the following inequalities hold,*

$$g(X_0, \mathcal{I}) \leq g(X, \mathcal{I}). \tag{12}$$

$$g((1+\rho))X, \mathcal{I}) \leq (1+\rho)g(X, \mathcal{I}). \tag{13}$$

Furthermore, if $A'XA \leq \alpha Q$, then

$$g((1+\rho)X, \mathcal{I}) \leq \left(1 + \frac{\rho}{1+\alpha}\right) g(X, \mathcal{I}). \tag{14}$$

Finally, the following Lemma provides a useful inequality regarding a positive semidefinite matrix and its trace.

Lemma 4. *Suppose that $X \in \mathbb{R}^{n\times n}$ is positive semidefinite, then*

$$X \leq \operatorname{tr}(X) I_n,$$

where $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix.

Proof. Since X is positive semidefinite, all the eigenvalues of X are no greater than $\operatorname{tr}(X)$, which concludes the proof. □

We are now ready to prove Theorem 1.

Proof (Theorem 1). Let us choose an arbitrary $\Sigma > 0$. First we will prove that $J(\sigma, \Sigma) \leq J(\sigma, \Sigma_0)$. To this end, define ρ_k as[6]

$$\rho_k \triangleq \inf\{\rho \geq 0 : \ (1+\rho)P_k(\sigma, \Sigma_0) \geq P_k(\sigma, \Sigma)\}.$$

By the definition of P_k,we have that

$$\begin{aligned} P_{k+1}(\sigma, \Sigma) &= g(P_k(\sigma, \Sigma), \mathcal{I}_{k+1}) \leq g((1+\rho_k)P_k(\sigma, \Sigma_0), \mathcal{I}_{k+1}) \\ &\leq (1+\rho_k) g(P_k(\sigma, \Sigma_0), \mathcal{I}_{k+1}) = (1+\rho_k)P_{k+1}(\sigma, \Sigma_0). \end{aligned}$$

The second inequality is true due to (13). Therefore, we know that $(1 + \rho_k)P_{k+1}(\sigma, \Sigma_0) \geq P_{k+1}(\sigma, \Sigma)$, which implies that $\rho_{k+1} \leq \rho_k$. As a result, ρ_k

[6] $\rho_k = 0$ if $P_k(\sigma, \Sigma_0) \geq P_k(\sigma, \Sigma)$.

is monotonically non-increasing. At this point it remains to prove that $\rho_k \to 0$. To this end, let us define $\alpha > 0$, such that

$$2J(\sigma, \Sigma_0)A'A \le \alpha Q.$$

Since we assume that $Q > 0$, we can always find such an α. Now from the definition of $J(\sigma, \Sigma_0)$, the following inequality holds infinitely often (i.e. for an infinite number of integers k):

$$\operatorname{tr}(P_k(\sigma, \Sigma_0)) \le 2J(\sigma, \Sigma_0).$$

By Lemma 4, we know that

$$A'P_k(\sigma, \Sigma_0)A \le A' \left[2J(\sigma, \Sigma_0)I_n\right] A \le \alpha Q, \tag{15}$$

infinitely often. Let k_i be a time index when (15) holds. By (14), we have

$$\rho_{k_i+1} \le \frac{1}{1+\alpha}\rho_{k_i}.$$

Since (15) happens infinitely often and $\alpha > 0$, it follows that $\rho_k \to 0$. At this point, we are ready to prove that $J(\sigma, \Sigma) \le J(\sigma, \Sigma_0)$. From the definition of J, we know that for all $k_0 \in \mathbb{N}$, the following equality holds

$$J(\sigma, \Sigma_0) = \limsup_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \operatorname{tr}(P_k(\sigma, \Sigma_0)) = \limsup_{N\to\infty} \frac{1}{N} \sum_{k=k_0}^{N} \operatorname{tr}(P_k(\sigma, \Sigma_0)).$$

Since ρ_k is non-increasing and

$$\operatorname{tr}(P_k(\sigma, \Sigma)) \le (1+\rho_k)\operatorname{tr}(P_k(\sigma, \Sigma_0)),$$

it follows that $J(\sigma, \Sigma) \le (1+\rho_k)J(\sigma, \Sigma_0)$ for all ρ_k. Now by using the fact that $\rho_k \to 0$, we prove that

$$J(\sigma, \Sigma) \le J(\sigma, \Sigma_0) < \infty.$$

By using the very same argument, one can also prove that $J(\sigma, \Sigma_0) \le J(\sigma, \Sigma)$. Therefore, for all $\Sigma > 0$, we can finally prove that

$$J(\sigma, \Sigma_0) = J(\sigma, \Sigma).$$

For the boundedness of schedule σ, since ρ_k is non-increasing, it is clear that

$$P_k(\sigma,\Sigma) \le (1+\rho_k)P_k(\sigma,\Sigma_0) \le (1+\rho_0)M,$$

which concludes the proof. □

4. Approximation of Feasible Schedule by Bounded Periodic Schedules

In this section, we prove that any feasible schedule can be arbitrarily approximated by bounded periodic schedules. The main result is summarized by the following theorem:

Theorem 5. *For any feasible schedule σ and for any $\varepsilon, \varepsilon_1, \ldots, \varepsilon_m > 0$, there exists a bounded periodic σ_p, such that*[7]

$$J(\sigma_p) \le J(\sigma) + \varepsilon, \tag{16}$$

and

$$r_i(\sigma_p) \le r_i(\sigma) + \varepsilon_i,\ i = 1, \ldots, m. \tag{17}$$

The following lemma is needed to prove Theorem 5.

Lemma 6. *Let σ_p be a periodic schedule with period $T > 0$, i.e.,*

$$\mathcal{I}_{k+T} = \mathcal{I}_k,\ \forall k.$$

If the following inequality holds for some initial condition $\Sigma > 0$

$$P_T(\sigma_p,\Sigma) \le \Sigma, \tag{18}$$

then the average cost function satisfies the following inequality,

$$J(\sigma_p) \le \frac{1}{T}\sum_{k=1}^{T} \operatorname{tr}(P_k(\sigma_p,\Sigma)). \tag{19}$$

Moreover, the schedule σ_p is bounded.

Proof. The main observation behind this proof is that, if the following inequality holds for any $k > 0$

$$P_{k+T}(\sigma_p,\Sigma) \le P_k(\sigma_p,\Sigma), \tag{20}$$

[7]For simplicity, we write $J(\sigma,\Sigma)$ as $J(\sigma)$ due to Theorem 1.

then (19) holds true. Interestingly enough, if we assume that (20) holds at a certain time $k = k_0$, then it will also hold true at all successive time steps. In fact:

$$\begin{aligned} P_{k_0+1+T}(\sigma_p, \Sigma) &= g(P_{k_0+T}(\sigma_p, \Sigma), \mathcal{I}_{k_0+1+T}) \leq g(P_{k_0}(\sigma_p, \Sigma), \mathcal{I}_{k_0+1+T}) \\ &= g(P_{k_0}(\sigma_p, \Sigma), \mathcal{I}_{k_0+1}) = P_{k_0+1}(\sigma_p, \Sigma), \end{aligned}$$

where we use the fact that σ_p is T-periodic. At this point, it is enough to remark that condition (18) is the inequality (20) evaluated at time $k = 0$, i.e.,

$$P_T(\sigma_p, \Sigma) \leq P_0(\sigma_p, \Sigma) = \Sigma,$$

to conclude that (18) implies (20), which finally implies (19). Please note that the latter arguments allow us to always find a scalar $M > 0$ such that

$$P_k(\sigma_p, \Sigma) \leq M,\ k = 1, \ldots, T.$$

By (20), we know that such an M is a uniform bound for all $P_k(\sigma_p, \Sigma)$. By Theorem 1, the schedule σ_p is bounded since $P_k(\sigma_p, \Sigma)$ is bounded for one initial condition Σ. □

Now we are ready to prove Theorem 5.

PROOF (THEOREM 5). By Theorem 1, J is independent of the initial condition Σ. As a result, let us choose $\Sigma = 2J(\sigma)I_n$. By the definition of J and r_i, there exists a $K > 0$, such that[8]

$$\frac{1}{N}\sum_{k=1}^{N} \operatorname{tr}(P_k(\sigma, \Sigma)) \leq J(\sigma) + \varepsilon, \tag{21}$$

$$\frac{1}{N}\sum_{k=1}^{N} \mathbb{I}_{s_i \in \mathcal{I}_k} \leq r_i(\sigma) + \varepsilon_i.\ i = 1, \ldots, m \tag{22}$$

for all $N \geq K$.

Furthermore, there must exist infinitely many scalars k, such that

$$\operatorname{tr}(P_k(\sigma, \Sigma)) \leq 2J(\sigma),$$

which implies that

$$P_k(\sigma, \Sigma) \leq 2J(\sigma)I_n = \Sigma. \tag{23}$$

[8]We denote $P_k(\sigma, \Sigma)$ as $P_k(\sigma)$ as Σ is fixed throughout the whole section.

As a result, we can choose T such that (21), (22) and (23) hold at the same time, i.e.,

$$\frac{1}{T}\sum_{k=1}^{T}\operatorname{tr}(P_k(\sigma,\Sigma)) \le J(\sigma)+\varepsilon, \tag{24}$$

$$\frac{1}{T}\sum_{k=1}^{T}\mathbb{I}_{s_i\in\mathcal{I}_k} \le r_i(\sigma)+\varepsilon_i.\, i=1,\dots,m \tag{25}$$

$$P_T(\sigma,\Sigma)\le\Sigma. \tag{26}$$

Now define a periodic schedule $\sigma_p = (\mathcal{J}_1, \mathcal{J}_2, \dots)$, such that

$$\mathcal{J}_{kT+j} = \mathcal{I}_j,\ \forall k\in\mathbb{N}_0,\ j=0,\dots,T-1.$$

Please note that the new schedule σ_p is the same as σ for the first T steps and then repeats itself afterwards. By definition, σ_p is periodic and (17) holds. By using Lemma 6, since

$$P_T(\sigma_p,\Sigma) = P_T(\sigma,\Sigma)\le\Sigma,$$

it follows that

$$J(\sigma_p)\le\frac{1}{T}\sum_{k=1}^{T}\operatorname{tr}(P_k(\sigma_p,\Sigma)) = \frac{1}{T}\sum_{k=1}^{T}\operatorname{tr}(P_k(\sigma,\Sigma))\le J(\sigma)+\varepsilon.$$

In addition, the schedule σ_p is bounded, which completes the proof. □

Remark 4. It is worth noticing that our proof is constructive and can be used to construct a periodic approximated schedule from any schedules.

5. Sensor Scheduling in Lossy Networks

In this section, we show how the properties stated in Theorem 1 and 5 hold true also in the case sensor scheduling over networks where sensor packets may be randomly dropped.

First, define $\gamma_{\mathcal{J},k}$, where $\mathcal{J}\subseteq S$, as a Bernoulli random variable such that $\gamma_{\mathcal{J},k}=1$ if at time k the set of indices of the received measurements is $\mathcal{J}$, and

$\gamma_{\mathcal{J},k} = 0$ otherwise. We assume that $\gamma_{\mathcal{J},k}$ is independent over time. Furthermore, we assume that the probability $P(\gamma_{\mathcal{J},k} = 1)$ only depends on the current set of selected sensors $\mathcal{I}_k$. As a result, we define the following probability

$$p_{\mathcal{J},\mathcal{I}} \triangleq P\left(\gamma_{\mathcal{J},k} = 1 | \mathcal{I}_k = \mathcal{I}\right). \tag{27}$$

The covariance P_k satisfies the following recursive equation:

$$P_k = \sum_{\mathcal{J}\subseteq S} \gamma_{\mathcal{J},k} g(P_{k-1}, \mathcal{J}),\ P_0 = \Sigma.$$

Thus, P_k is random. Similar to [25], we can derive the upper bound V_k of $\mathbb{E}P_k$ as

$$V_k = \sum_{\mathcal{J}\subseteq S} p_{\mathcal{J},\mathcal{I}} g(V_{k-1}, \mathcal{J}),\ V_0 = \Sigma.$$

Now define $\tilde{g}(X, \mathcal{I})$ as

$$\tilde{g}(X, \mathcal{I}) \triangleq \sum_{\mathcal{J}\subseteq S} p_{\mathcal{J},\mathcal{I}} g(X, \mathcal{J}).$$

Therefore

$$V_k = \tilde{g}(V_{k-1}, \mathcal{I}),\ V_0 = \Sigma.$$

At this point, it is enough to note that $\tilde{g}$ satisfies all the properties in Lemma 3. Hence, by simply replacing P_k with V_k, all the results in Section 3 and Section 4 hold true also in this case.

6. Numerical Example

In this section, we consider a simple scalar system for which both the *infinite-horizon* optimal schedule and the optimal *periodic* schedule can be derived:

$$x_{k+1} = x_k + w_k,$$

with $\Sigma = Q = 1$. We further assume that only one sensor is measuring the state, i.e.,

$$y_k = x_k + v_k.$$

We consider an extreme case by assuming that the sensor is perfect. In other words, the covariance of the measurement noise $R = 0$. We further assume that $\mathcal{S} = \{\emptyset, \{1\}\}$. In this particular case, the g function can be simplified as

$$g(X, \mathcal{I}) = \begin{cases} X + 1 & \text{if } \mathcal{I} = \emptyset \\ 0 & \text{if } \mathcal{I} = \{1\} \end{cases}. \tag{28}$$

Given a schedule σ, let us define the set $\mathcal{E}(\sigma)$ as

$$\mathcal{E}(\sigma) \triangleq \{k \in \mathbb{N} : \mathcal{I}_k = \{1\}\}.$$

Hence, $\mathcal{E}(\sigma)$ is the set of the time indices when the sensor is selected.

Consider the following optimization problem:

$$\begin{aligned} &\underset{\sigma}{\text{minimize}} && J(\sigma) \\ &\text{subject to} && r_1(\sigma) \le r, \end{aligned} \tag{29}$$

where $r \geq 2/3$. The following theorem provides a lower bounds for (29)

Lemma 7. *The optimal objective function J^* of* (29) *satisfies*

$$J^* \geq 1 - r.$$

Proof. For an arbitrary schedule σ satisfying $r_1(\sigma) \leq r$, consider the finite average

$$J_N(\sigma) = \frac{1}{N} \sum_{k=1}^{N} P_k.$$

By (28), if $k \in \mathcal{E}(\sigma)$, then $P_k = 0$. Otherwise, $P_k \geq 1$. Define $\mathcal{E}_N(\sigma)$ as

$$\mathcal{E}_N(\sigma) \triangleq \mathcal{E}(\sigma) \bigcap \{1, \dots, N\}.$$

As a result,

$$J_N(\sigma) \geq \frac{1}{N} \times (N - |\mathcal{E}_N(\sigma)|) = 1 - \frac{|\mathcal{E}_N(\sigma)|}{N},$$

where $|\cdot|$ indicates the cardinality of a set. By taking limit on both sides, we have

$$J(\sigma) = \limsup_{N \to \infty} J_N(\sigma) \geq 1 - r,$$

which completes the proof. $\square$

Consider a periodic schedule of length T. The following theorem characterize the optimal periodic schedule:

Lemma 8. *For any periodic schedule σ with period $T \geq 2$ and satisfies $r_1(\sigma) \leq r$, the following inequality holds*

$$J(\sigma) \geq 1 - \frac{\lfloor rT \rfloor}{T}. \tag{30}$$

where $\lfloor \cdot \rfloor$ denotes the standard floor operator. Furthermore, if $r \geq 2/3$, then equality is achieved for the following periodic schedule:

$$(\mathcal{I}_1, \dots, \mathcal{I}_T) = (\underbrace{\{1\}, \dots, \{1\}}_{2\lfloor rT \rfloor - T}, \underbrace{\{1\}, \emptyset, \{1\}, \emptyset, \dots, \{1\}, \emptyset}_{2T - 2\lfloor rT \rfloor}). \tag{31}$$

Proof. (30) can be proved by the same argument as presented in the proof of Lemma 7. It is easy to verify that (31) satisfies the equality in (30). □

Since any real number r can be arbitrarily approximated by a rational number $\lfloor rT \rfloor / T$, we have the following theorem:

Theorem 9. *If $r \geq 2/3$, then the optimal objective function J^* of (29) is given by*

$$J^* = 1 - r.$$

Figure 1 shows the optimality gap of optimal periodic schedule versus the period T.

Remark 5. It can be seen that, given our choice of system parameters, the optimality gap depends on how good a rational approximation of r is. Since the rational number is dense in $\mathbb{R}$, we can find a periodic schedule, the performance of which is arbitrarily close to the optimal performance, given that the period length is large enough. However, as is shown in Fig 1, the relationship between the optimality gap and period length is non-monotonic and in general quite involved.

7. Conclusion

In this paper, we consider the problem of infinite-horizon sensor scheduling problem for linear Gaussian systems. We assume that at each time step only a subset of all sensors can be selected to send their observations to the fusion center. We prove that the infinite-horizon cost function and the boundedness

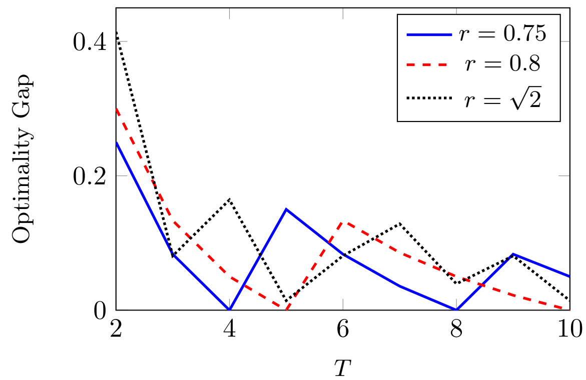

Figure 1: Optimality Gap of the Optimal Periodic Schedule versus Period Length

of a schedule are independent of the initial covariance matrix. We further prove that any feasible schedule can be arbitrarily approximated by a periodic schedule. The proof of the latter result is constructive and it thus provides useful insights into the design of computationally efficient periodic approximation with quantifiable sub-optimality. The results are then extended to lossy networks. To give to the reader some insight on the nature of the performance attainable with periodic schedules, the results of the paper are particularized to a simple scalar system for which both the *infinite-horizon* optimal schedule and the optimal *periodic* schedules can be computed. Future research activities will aim at investigating the non-monotonic complex relationships between the performance of periodic scheduling and the length of the period.

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